ALGEBRA

Formulae →

(1) $(a+b)^2 = (a-b)^2 + 4ab$

(2) $(a-b)^2 = (a+b)^2 - 4ab$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(5) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(6) $(a^3 - b^3) = (a-b)(a^2 + b^2 + ab)$

(7) $(a^3 + b^3) = (a+b)(a^2 + b^2 - ab)$

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(8) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(9)* $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(10)* or $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}[(a+b+c)][(a-b)^2 + (b-c)^2 + (c-a)^2]$

Note → $a^3 + b^3 + c^3 - 3abc = 0$,

If (i) $a+b+c = 0$ $(a \neq b \neq c)$ - or

(ii) $a^2 + b^2 + c^2 - ab - bc - ca = 0$ $(a = b = c)$

(11)* $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$

(12) If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(13)* If $(ax + by) = m$ and $bx - ay = n$, then

(i) $(a^2 + b^2)(x^2 + y^2) = m^2 + n^2$

(ii) $(a^2 - b^2)(x^2 - y^2) = m^2 - n^2$

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(14) $a^4 + a^2b^2 + b^4 = (a^2+ab+b^2)(a^2-ab+b^2)$

(15)* (i) If $a+b+c = X$, and $a^2+b^2+c^2 = Y$, then

$$a^3+b^3+c^3 - 3abc = \frac{3XY - X^3}{2}$$ Ans.

(ii) If $a^2+b^2+c^2 = X$, and $ab+bc+ca = Z$

$$a^3+b^3+c^3 - 3abc = X^3 - 3XZ$$

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(16)* (i) If a, b, c are three consecutive no. ($a=1, b=2, c=3$) →

$$a^2+b^2+c^2 - ab - bc - ca = 3 \times (diff)^2$$

(ii) If $a = b$ & $c = a+n$ and $= b+n$ → $c-a = cd$ & $c-b = cd$ (common difference)

Let $a = b = 2$, $c = 4$. Then $[c-a = c-b = 4-2 = 2]$

$$a^2+b^2+c^2 - ab - bc - ca = (diff)^2$$

(iii) If a, b, c are non consecutive no., then ($a=2, b=7, c=18$)

$$a^2+b^2+c^2 - ab - bc - ca = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$$ → Basic formula.

17* (i) If a, b, c are three consecutive no., then

$$a^3 + b^3 + c^3 - 3abc = 9 \times b \times (diff)^2$$

(ii) If $a = b$ & $c-a = c-b = cd$, then

$$a^3 + b^3 + c^3 - 3abc = (a+b+c) \times (diff.)^2$$

(iii) If a, b, c are non consecutive no., then

$$a^3+b^3+c^3 - 3abc = \frac{(a+b+c)}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

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(iv) If $a+b+c=0$ Then →

$$\boxed{a^3+b^3+c^3 = 3abc}$$

(18.)* (i) If $a+b+c=0 \Rightarrow \boxed{a=1,\ b=1,\ c=-2}$ Put.

(ii) If $pq+qr+rp=0 \Rightarrow \boxed{p=2,\ q=2,\ r=-1}$ Put.

(iii) If $xy=1 \Rightarrow$ Then $\boxed{\frac{1}{x+1}+\frac{1}{y+1}=1}$

(19.)* How to calculate value of x, y, and z in a quadratic equⁿ →

$$\Rightarrow x = -\frac{1}{2}\left[\frac{\text{coefficient of } x}{\text{coefficient of } x^2}\right]$$

$$\Rightarrow y = -\frac{1}{2}\left[\frac{\text{coeff. of } y}{\text{coeff. of } y^2}\right]$$

$$\Rightarrow z = -\frac{1}{2}\left[\frac{\text{coeff. of } z}{\text{coeff. of } z^2}\right]$$

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(20.) $a^3+b^3-c^3+3abc = (a+b-c)(a^2+b^2+c^2-ab+bc+ca)$

(21.) $a^3-b^3-c^3-3abc = (a-b-c)(a^2+b^2+c^2+ab+ca-bc)$

(22.)* If $P=a$, then,

(i) $\sqrt[3]{P(P^2+3P+3)+1} = a+1$

(ii) $\sqrt[3]{P(P^2+3P+3)-1} = a-1$

(iii) $P(P^2+3P+3) = (a+1)^3-1$

(iv) $P(P^2-3P+3) = (a-1)^3+1$

(23.) $(a+b-c)^2 = a^2+b^2+c^2+2(ab-bc-ca)$

(24.)* $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

(25.)* $(a+b)^2-(a-b)^2 = 4ab$

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(1.) $x + \frac{1}{x} = a \;, \Rightarrow \quad x^2 + \frac{1}{x^2} = a^2 - 2$

(2.) $x + \frac{1}{x} = \sqrt{a} \;, \Rightarrow \quad x^2 + \frac{1}{x^2} = a - 2$

(3.) $x - \frac{1}{x} = a \;, \Rightarrow \quad x^2 + \frac{1}{x^2} = a^2 + 2$

(4.) $x - \frac{1}{x} = \sqrt{a} \;, \Rightarrow \quad x^2 + \frac{1}{x^2} = a + 2$

(5.) $x + \frac{1}{x} = a \;, \Rightarrow \quad x - \frac{1}{x} = \sqrt{a^2 - 4}$

(6.) $x + \frac{1}{x} = \sqrt{a} \;, \Rightarrow \quad x - \frac{1}{x} = \sqrt{a - 4}$

(7.) $x - \frac{1}{x} = a \;, \Rightarrow \quad x + \frac{1}{x} = \sqrt{a^2 + 4}$

(8.) $x - \frac{1}{x} = \sqrt{a} \;, \Rightarrow \quad x + \frac{1}{x} = \sqrt{a + 4}$

(9.) $x - \frac{1}{x} = a \;, \Rightarrow \quad x^2 - \frac{1}{x^2} = a\sqrt{a^2 + 4}$

(10.) $x - \frac{1}{x} = \sqrt{a} \;, \Rightarrow \quad x^2 - \frac{1}{x^2} = \sqrt{a}\sqrt{a + 4}$

(11.) $x + \frac{1}{x} = a \;, \Rightarrow \quad x^2 - \frac{1}{x^2} = a\sqrt{a^2 - 4}$

(12.) $x + \frac{1}{x} = \sqrt{a} \;, \Rightarrow \quad x^2 - \frac{1}{x^2} = \sqrt{a}\sqrt{a - 4}$

(13.) $x + \frac{1}{x} = a \;, \Rightarrow \quad x^3 + \frac{1}{x^3} = a^3 - 3a$

(14.) $x + \frac{1}{x} = \sqrt{a} \;, \Rightarrow \quad x^3 + \frac{1}{x^3} = \sqrt{a}(a - 3)$

(15.) $x - \frac{1}{x} = a \;, \Rightarrow \quad x^3 - \frac{1}{x^3} = a^3 + 3a$

(16.) $x - \frac{1}{x} = \sqrt{a} \;, \Rightarrow \quad x^3 - \frac{1}{x^3} = \sqrt{a}(a + 3)$

(17.) $x + \frac{1}{x} = a \;, \Rightarrow \quad x^3 - \frac{1}{x^3} = (a^2 - 1)\sqrt{a^2 - 4}$

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(18.) $x + \frac{1}{x} = \sqrt{a}$, $\Rightarrow x^3 \frac{1}{x^3} = (a-1)\sqrt{a-4}$

(19.) $x - \frac{1}{x} = a$, $\Rightarrow x^3 + \frac{1}{x^3} = (a^2+1)\sqrt{a^2+4}$

(20.) $x - \frac{1}{x} = \sqrt{a}$, $\Rightarrow x^3 + \frac{1}{x^3} = (a+1)\sqrt{a+4}$

(21.) $x^4 - \frac{1}{x^4} = a$, (i) $x^2 + \frac{1}{x^2} = \sqrt{a+2} = b$ (Let)

(ii) $x + \frac{1}{x} = \sqrt{b+2}$

(22.) $x^4 + \frac{1}{x^4} = a$, (iii) $x - \frac{1}{x} = \sqrt{b-2}$

(23.) Same as above.

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If $x + \frac{1}{x} = 1$, then $\longrightarrow$ $\boxed{x^3 = -1}$ Put

(24.) If $x + \frac{1}{x} = \sqrt{3}$, or $x^2 + \frac{1}{x^2} = 1$, Then $\boxed{x^6 = -1}$ Put

(25.)* If $x + \frac{1}{x} = 1$, (i) Then put, $x^n = -1$ for odd power

(ii) Put, $x^n = 1$ for even power

Note → Power should be multiple of 3.

(26.) If $x + \frac{1}{x} = -1$, Then $\boxed{x^3 = 1}$ Put.

(i) Put $x^6 = -1$, for odd power

(ii) Put $x^6 = +1$, for even power

Note → Power should be multiple of 6.

(27.) find Minimum value of x in quadratic equn →

let $(x \pm a)(x \pm b) = 0$, Then for minimum value →

$\Rightarrow \pm a - (\pm b) = t$

for $\boxed{\text{minimum value} = -\frac{1}{4}(\pm t)^2}$ or $\frac{-b^2}{4ac}$ where a & b are any integer no.

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(28.) $\frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{2(x^2+y^2)}{(x^2-y^2)}$

(29.) $\frac{x+y}{x-y} - \frac{x-y}{x+y} = \frac{4xy}{(x^2-y^2)}$

(25.) If $x = a+b$ is a no. [for eg. $x = 3+2\sqrt{2}$] and difference of square of a and b give 1.

Then →

$x + \frac{1}{x} = 2a$ [If $a>b$ and $a^2-b^2=1$]

$x + \frac{1}{x} = 2b$ [if $b>a$ and $b^2-a^2=1$]

$x - \frac{1}{x} = 2b$ [if $a>b$ and $a^2-b^2=1$]

$x - \frac{1}{x} = 2a$ [if $b>a$ and $b^2-a^2=1$]

(26.) if $\sqrt{x\sqrt{x\sqrt{x\sqrt{x}\ldots\infty}}} = x$ Values of x →

(27.) if $\sqrt{x\sqrt{x\sqrt{x\sqrt{x}\ldots}}}$ n Times $= x^{\left(\frac{2^n-1}{2^n}\right)}$

(28.) if $\sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}} = \frac{\sqrt{4x+1}+1}{2}$

(29.) if $\sqrt{x-\sqrt{x-\sqrt{x-\ldots\infty}}} = \frac{\sqrt{4x+1}-1}{2}$

(30) If $ax+by=c$, $bx-ay=d$, Then →

$\Rightarrow (a^2+b^2)(x^2+y^2) = (c^2+d^2)$

If $ax+by=c$, $bx+ay=d$, Then →

$\Rightarrow (a^2+b^2)(x^2-y^2) = (c^2-d^2)$

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Some Special Results:→

(1.) If $x+\frac{1}{x}=a$, Then

(i) $x^5+\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$

or $\boxed{x^5+\frac{1}{x^5}=(a^2-2)(a^3-3a)-a}$

(ii) If $x-\frac{1}{x}=a$, Then

$x^5-\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3-\frac{1}{x^3}\right)-\left(x-\frac{1}{x}\right)$

or $\boxed{x^5-\frac{1}{x^5}=(a^2+2)(a^3+3a)-a}$

(2.) (i) If $x+\frac{1}{x}=a$, Then

$x^6+\frac{1}{x^6}=\left(x^3+\frac{1}{x^3}\right)\left(x^3+\frac{1}{x^3}\right)-2$

or $\boxed{x^6+\frac{1}{x^6}=(a^3-3a)^2-2}$

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(3.) (ii) If $x+\frac{1}{x}=a$, Then

$x^7+\frac{1}{x^7}=\left(x^4+\frac{1}{x^4}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$

or $\boxed{x^7+\frac{1}{x^7}=[(a^2-2)^2-2]\times(a^3-3a)-a}$

(ii) If $x-\frac{1}{x}=a$, Then

$x^7-\frac{1}{x^7}=\left(x^4+\frac{1}{x^4}\right)\left(x^3-\frac{1}{x^3}\right)+\left(x-\frac{1}{x}\right)$

or $\boxed{x^7-\frac{1}{x^7}=[(a^2+2)^2-2]\times(a^3+3a)+a}$

Note→ a की Power या whole power के साथ जो भी (+) या (−) में दिया होता है, उसके साथ $\left(\text{यानि } \pm\frac{1}{x}\right)$ x वाले $\frac{1}{x}$ के coeff. multiple में हमेशा उसी है।

Ex- If $7x+\frac{1}{2x}=14$, Then find $49x^2+\frac{1}{4x^2}=?$

∵ $x+\frac{1}{x}=a$, Then $x^2+\frac{1}{x^2}=a^2-2\cdot x\text{ coeff}\times x\text{ coeff of }\frac{1}{x}$ so → $=14^2-2\times7\times\frac{1}{2}$

$=189$ Ans

This is apply for all from formula (1.) to (22.) and 1 to 3. (ii)

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(1.) How to find Minimum and Maximum Values

(2.) If small Power with -ve Sign Then →

(i) a^3+4a^2-a , Eg If $x = 2-2^{1/3}+2^{2/3}$ Then find $x^3-6x^2+18x = ?$ where $a=2$

(ii) When greater Power with -ve Sign Then →

a^3+2a^2+a , eg- If $x = 2-2^{2/3}+2^{1/3}$, Then find $x^3-6x^2+18x = ?$ where $a=2$

(iii) When all Power with +ve Sign Then →

a^3-2a^2+a, eg = If $x = 5+5^{1/3}+5^{2/3}$ Then find $x^3-15x^2+60x = ?$ where $a=5$

(2.) If

	Max	Min
x	$+\infty$	$-\infty$
x^2	$+\infty$	0
x^3	$+\infty$	$-\infty$

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(3.) If m is a positive Real No → $m, \frac{1}{m}$ Then

$AM \geq GM$

$m+\frac{1}{m} \geq 2$ for minimum value

(4.) If $ax^2+bx+c=0$

If $a \rightarrow +ve$ Max $\rightarrow +\infty$ Min $\rightarrow \frac{4ac-b^2}{4a}$

If $a \rightarrow -ve$ Max $\rightarrow \frac{4ac-b^2}{4a}$ Min $\rightarrow -\infty$

(5.) If $x+y$ given, $x \cdot y \Longleftrightarrow$ Max. when $x=y$

Ex- If $x>1$ find the min value of →

$$x^2+\frac{1}{x^2-1}-3 = ?$$

Solu $(x^2-1)+\frac{1}{(x^2-1)}-3+1 \Rightarrow 2-3+1 \Rightarrow 0$ Ans

↳ 2 min value

इस तरह के Q. में हम $m+\frac{1}{m}$ जैसी Shape बनाते हैं

अपनी तरफ से उतना ही add and substract करेंगे

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If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{x}{y}$

Then (i) $a = c = e = x$ and $b = d = f = y$

(ii) $\frac{a-c}{b-d} = \frac{c-e}{d-f} = \frac{e-a}{f-b}$

(iii) $\frac{a+c}{b+d} = \frac{c+e}{d+f} = \frac{e+a}{f+b}$

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If $pqr = 1$, Then $p = q = r = 1$

Q. $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$ Then find $\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} = ?$

Solun जब हम इस तरह के Q. मे value रखे कर Solve ना कर पाये तो हम step को छोटा कर लेते हैं। किन्ही दो की value रख कर और तीसरे को as a tease रख कर →

Put $a = 0$, $b = 1$

$0 + \frac{1}{c+0} + \frac{c}{0+1} = 1$ — ①

or $c^2 = c - 1$ — ②

find $- \frac{0}{1+c} + \frac{1}{c+0} + \frac{c^2}{0+1}$

$\frac{1}{c} + c^2$ $[\because c^2 = c-1]$

$\frac{1}{c} + c - 1$ $[\because \frac{1}{c} + c = 1]$

$1 - 1 = 0$ Ans

Q. $\frac{x}{x+1} + \frac{x}{x+2} + \frac{x}{x+3} + \cdots \frac{x}{x+1009} = 1$ Then find →

$\frac{1}{x+1} + \frac{2}{x+2} \cdots\cdots \frac{1009}{x+1009} = ?$

Trick - इस तरह के Q. मे हम, जो find करना है सबसे पहले उस form मे लाने की कोशिश करते हैं → By two Method →

(1) या तो find वाली form नीचे वाले मे ऊपर वाला जोड़कर बनेगी या

(2) नीचे वाले मे से ऊपर वाला घटाकर बनेगी — $(x+1)(x+2)$ —)

अगर जोड़ने से बने तब → (Total no. of terms + equal के उसके side वाली value)

अगर घटाने से बने तब → (Total no. of terms + equal वाली value)

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Eg-1 If $\frac{x}{x+1} + \frac{x}{x+2} + \frac{x}{x+3} + \cdots\cdots \frac{x}{x+1009} = 1$ ← equal वाली value.

Then find $\frac{1}{x+1} + \frac{2}{x+2} + \cdots\cdots \frac{1009}{x+1009} = ?$

Solun

$$\frac{x-(x+1)}{x+1} + \frac{x-(x+2)}{x+2} + \cdots\cdots \frac{x-(x+1009)}{x+1009} = 1-1009$$

$$= -\frac{1}{x+1} - \frac{2}{x+2} - \frac{3}{x+3} \cdots\cdots - \frac{1009}{x+1009} = 1-1009$$

$$\frac{1}{x+1} + \frac{2}{x+2} + \frac{3}{x+3} \cdots\cdots \frac{1009}{x+1009} = 1008 \text{ Ans}$$

Trick → $\because \frac{x-(x+1)}{(x+1)} = \frac{-1}{x+1}$ ∵ हम पता चल रहा है कि यहाँ घटोनेसे ve no form बन रही है तब,

Total no. of terms - equal वाली value

$1009 - 1 = 1008$ Ans

Eg.2 If $\frac{a^2+ab}{a^2-ab} + \frac{b^2+cd}{b^2-cd} + \frac{c^2+de}{c^2-de} = 5$ Then find $\frac{a^2}{a^2-ab} + \frac{b^2}{b^2-cd} + \frac{c^2}{c^2-de} = ?$

Trick $\frac{a^2+ab+(a^2-ab)}{a^2-ab} = \frac{2a^2}{a^2-ab}$ → isko common le lenge

Total no. of term + equal me jo hai

$$\frac{3+5}{\text{common value}} = \frac{8}{2} = 4 \text{ Ans}$$

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If $x = \sqrt{a^3\sqrt{a^3\sqrt{a^3\sqrt{\cdots\infty}}}} = \sqrt[5]{a^3 b}$

If $x^n + \frac{1}{x^n} = k \Rightarrow x^n \pm \frac{1}{x^n} = \pm\sqrt{k^2 \pm 4}$

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